ME256 Jan. 3:0

Variational Methods and Structural Optimization

Instructor

G. K. Ananthasuresh
Email: suresh@iisc.ac.in

Teaching Assistant

Prasenjit Ghosh
Email: prasenjitg@iisc.ac.in

Department: Mechanical Engineering

Course Time: Tue., Thu., 8:30 - 10:00 AM

Lecture venue: Multimedia Classroom in Mechanical Engineering

Detailed Course Page: http://www.mecheng.iisc.ernet.in/~suresh/me256/

Announcements

The first lecture of the 2018 session is at 8:30 AM on January 4, 2018 in ME MMCR.

Brief description of the course

This course is suitable to engineering students of most disciplines where continuous models are used. It includes detailed discussion of calculus of variations and introduction to analytical and computational methods of structural optimization.

Background to the course

Optimization is a way to get the best out of what is available. It is thus no wonder that everybody tries to optimize almost everything. Even in Nature, it appears as if everything is optimized based on some criteria and subject to some constraints. What then are the mathematical tools that help us analyze and obtain such optima? Variational methods, or more precisely the calculus of variations, is a primary mathematical tool that helps us in this regard.

While the ordinary calculus considers functions of finite number of variables, the calculus of variations, a phrase coined by Euler, considers functions of functions themselves. That is, in optimization of functions of finite number of variables of ordinary calculus, we find minimizing values of such finite number of variables whereas in calculus of variations, we find the minimizing function on which another function (called a functional) depends. What the derivative is to ordinary calculus the variation is to calculus of variations. There are subtle similarities and profound differences between the two.

The recorded scieintific history shows that ancient Greeks had formulated some problems that now fall under calculus of variations. Galileo had considered a few problems of which one, the Brachstochrone problem, has become the most popular. Fermat put forth a variational problem for the refraction of light rays saying they follow the path of least time and not the least distance. Newton had considered the surface of revolution of a solid that experiences least resistance when traveling in a fluid along its axis. John and James Bernoullis, Leibnitz, L'Hopital, Newton, and others revived the Brachistochrone problem. Euler and Lagrange laid the firm analytical foundation for it. Lagrange, Hamilton, Jacobi and others used it in mechanics. Legendre, Jacobi and others worked further on calculus of variations.

Structures are continuous. More recent studies on compliant mechanisms show that they are also very much like structures except that they are flexible. Both the governing equations of equilibrium of structures as well as the methods of their optimum shape and topology can be cast as variational problems.

This course will be an exciting journey from Fermat's principle of least time to a practical implementation of topology optimization of structures and compliant mechanisms--a journey from the classical to the contemporary times.

And, remember that optimization hinders evolution! Nature takes many, many years to evolve to an optimum whereas engineers design optimum structures as fast as the computers can churn the numbers.

Prerequisites

Multivariable calculus; familiarity with elastic mechanics and finite element analysis is preferred.

Syllabus

Module 1

Motivating examples of calculus of variations

Mathematical preliminaries: normed vectors spaces, functionals (continuous and linear), directional derivative, of E-L equation to multiple derivatives, independent variables, multiple state variables

Isoperimetric problems--global and local (finite subsidiary) constraints

Applications of optimizing functionals subject to constraints

Applications in mechanics: strong and weak forms of governing equations

Variable end conditions--transversality conditions

Module 2

Size optimization of a bar for maximum stiffness

Self-adjointness and optimization with weak variational form

Optimization with side constraints (variable bounds)

Worst load scenario for an axially loaded stiffest bar

Min-max type problem with stress constraints

Beam problems for stiffmess and strength

Optimization of a beam for given deflection

Variational formulations for the eigenvalue problems: strings, bars, beams, and other elastic strucrures.

Optimum design of a column

Variable-thickness optimization of plates

Module 3

Truss topology optimization

Sensitivity analysis

Frame topology optimization

Compliant mechanism design using topology optimization of trusses and frames

Topology optimization using continuum elements

Optimality criteria method

Shape optimization of structures

Applications to multi-physics problems

Material interpolation techniques for topology optimization

Course outcomes

After taking this course, a student would...

1) Understand the difference between ordinary calculus and calculus of variations as well as functions and functionals.

2) Get a quick grasp of the terminology of function spaces, energy spaces in particular.

3) Be able to take the first variation of a functional.

4) Write down necessary conditions of functionals involving multiple functions; multiple derivatives of a function; one, two, or three independent variables on which the functions depend.

5) Understand how to write the boundary conditions, including variable end conditions and transversality conditions.

6) Appreciate energy and variational methods in mechanics as well as the interconnection between force-balance (differential equation), weak form (principle of virtual work and D' Lambert principle), and energy principles (minimum potential energy and Hamilton's principle) in mechanics.

7) Be able to think about the inverse problem of writing the minimization principle from the differential equation.

8) Gain a thorough understanding of Karush-Kuhn-Tucker (KKT) conditions for constrained minimization problems and the concept of Lagrange multipliers and their various interpretations.

9) Be able to analytically obtain the necessary conditions for optimizing a bar of variable cross-section profile for different objective functions and constraints.

10) Be able to the same for beams, plates, 2D and 3D continuous structures.

11) Understand the sensitivity analysis in structural optimization.

12) Be able to implement the numerical optimization algorithm to obtain optimized geometry of bars, beams,

plates, 2D continua, 2D and 3D trusses, 2D and 3D frames and grillages.

13) Be able to consider transient and multiphysics problems in structural optimization.

14) Become familiar with Optimization Toolbox in Matlab.

15) Be able to formulate optimization problems in the framework of calculus of variations and then convert into the discretized form as a finite-variable continuous optimization.

Grading policy

25% for assignments (about 10 in the semester)

25% for the mid-term examination

25% for a course project (individual project done in the later half of the semester)

25% for the final examination

Assignments

A homework assignment of about three problems will be given, roughly every week barring the weeks when there is an examination or when the project is due.

Some problems in the assignments are deliberately open ended to challenge the student and to encourage critical thinking on how to formulate optimization and optimal design problems and solve them.

All assignments are posted online at the detailed course page at

http://www.mecheng.iisc.ernet.in/~suresh/me256/homework.html

Resources

Textbook

There is no prescribed textbook for this course. The principal course material will be the notes taken during the lecture. Some handouts will be given. Papers from the contemporary literature will be provided. Some reference books are listed below.

Primary reference books

Robert Weinstock "Calculus of Variations with Applications to Physics and Engineering", now in Dover publications in paperback form.

It is a comprehensive book with an optimum balance between rigour, conceptual understanding, and emphasis on applications.

A. S. Gupta, "Calculus of Variations with Applications", Prentice-Hall of India Pvt. Ltd., New Delhi, 2008.

A very accessible book from the viewpoints of subject mattter and price. Among other things, it contains a good discussion of the sufficient conditions for the calculus of variations problems.

I. M. Gelfand and S. V. Fomin "Calculus of Variations", now in Dover publications in paperback form.

It is one of the classical books on calculus of variations. It starts from the very basics and goes to an advanced level. It is even suitable for self-study.

This book is kept on reserve in the J. R. D. Tata library in the reference section so that you all can refer to it in the library and may borrow it for over-night study as per the rules of the library.

Smith, D. R., "Variational Methods in Optimization," Dover Publications, 1998.

It gives a comprehensive treatment of calculus of variations starting from the basics. Many examples are included. Although it is written by a mathematician, it is rich in applications. It also gives sufficient insight into the mathematical concepts.

This book is kept on reserve in the J. R. D. Tata library in the reference section so that you all can refer to it in the library and may borrow it for over-night study as per the rules of the library.

Haftka, R. T. and Gurdal, Z., "Elements of Structural Optimization," Kluwer Academic Publishers, 1992.

The book gives a nice exposition of classical structural optimization. It has some examples that use variational methods approach. It also included numerical optimization techniques.

Bendsoe, M. P. and Sigmund, O., "Topology Optimization: Theory, Methods, and Applications," Springer, 2003.

A contemporary book on topology optimization. It is rich in methods and includes some examples. Its bibliography is very useful as well.

Other reference books

David G. Luenberger, "Optimization by Vector Space Methods," John-Wiley & Sons, 1969.

This is an excellent book for getting a rigorous and intuitive understanding of vector spaces and functionals. It gives a lucid treatment of such important concepts as Hahn-Banach theorem, etc.

P. Y. Papalambros and D. J. Wilde "Principles of Optimal Design," Cambridge University Press, 2000.

This is a great book if someone wants to learn about optimal design, optimization theory, and applications all from a single book. The monotonicity analysis presented in this book makes it extra special.

Herman H. Goldstein, "A History of the Calculus of Variations: from the 17th through 19th century," Springer-Verlag, 1980.

An authoratative exposition of the history of calculus of varitions from Fermat/Galileo to Wierstrass and Du Bois Raymond and others.

Journals

You can learn about the application of optimization to the design problems by reading articles from the following journals.

Journal of Mechanical Design

Engineering Optimization

Structural and Multidisciplinary Optimization

Journal of Computational Optimization and Applications

Research in Engineering Design

Journal of Mechanics of Machines and Structures

Finite Elements in Analysis and Design

Web sites on optimization in general

Check this out; it is a very useful reference to have at your finger tips -- quite literally with the mouse and keynboard interface. Mathematical Programming Glossary

Multidisciplinary Optimization Branch at NSA Langley Research Center.

A general resource to mathematical software.

Use the search engine at this site to look for software on optimization.

Vanderplaats Research and Development

We have the DOC/DOT software developed by this company. You might it useful in your project work. Note that this software is written in FORTRAN.

Professor Tom Cavalier's (Penn State) Optimization Links.

External Sources of Numerical Optimization Information.

Optimization Technology Center run by North Western University and Argonne National Labs.

NEOS (Network Enabled Optimization System Guide.

Quint Inc. that markets topology optimization software.

An interesting math history site with a lot of useful information is here.

Numerical recipes are here!.

MSC Software's optimization site is here .

Altair Engineering's site. Look at HyperShape/Pro.

Some university sites

Professor Papalambros' research group at the University of Michigan.

Professor Azarm's site at University of Maryland.

Stanford University's optimization group

derived the same independently in 1948. That work was rejected for a

journal and appeared elsewhere. Works of Karush and John didn't attract the attention but that of Harold Kuhn and Alan Tucker, done in 1951 at Princeton, did. Kuhn and Tucker also had derived this independently but gave due credit to Karush. So, the conditions are now known to be Karush-Kuhn-Tucker conditions or KKT conditions.

Here is an obituary for Karush published in the University of Chicago magazine in October 1997.
William Karush, SB'38, SM'39, PhD'42, a research scientist and a professor emeritus of mathematics at California State University, Northridge, died February 22 in Los Angeles. He was 79. Karush was one of 37 Manhattan Project scientists to sign a 1945 petition urging the American government to restrain its use of nuclear weaponry, and he traveled internationally during the cold war as an activist for peaceful resolution to global conflict. Karush worked as a research scientist for Ramo-Wooldridge Corp. (now TRW) before becoming principal scientist at System Development Corp. in Santa Monica, CA, where he later headed system-sciences research. He wrote Webster's New World Dictionary of Mathematics. Survivors include his companion, Hope Wallace; a son; a daughter; a brother; a sister; and three grandchildren.